CORE MATHEMATICS (C) UNIT 1 TEST PAPER 9

1. Find, in its simplest form, the exact value of $\dfrac{3}{2+2\sqrt{2}} - \dfrac{3}{\sqrt{2}}$. [4]

2. Differentiate with respect to x:

   (i) $(2x + 3)(2 - 3x)$, (ii) $(\sqrt{x})^3$. [5]

3. The vertices of a triangle are $P\,(-2, 2)$, $Q\,(0, 6)$ and $R\,(8, t)$.

   Given that PQ is perpendicular to QR, find

   (i) the value of t, [3]

   (ii) the area of triangle PQR. [3]

4. (i) Given that $x = \dfrac{1}{y^2}$, express $\dfrac{18}{y^4}$ in terms of x. [1]

   (ii) Hence find the real value of y for which $\dfrac{18}{y^4} + \dfrac{1}{y^2} = 4$. [5]

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6. Given that $f(x) \equiv (x + 3)^2$, sketch the following graphs on separate diagrams. In each case show the coordinates of the minimum point and any points where the graph intersects the x and y axes.

(i) $y = f(x)$, [2]

(ii) $y = 2f(x)$, [2]

(iii) $y = f(x - 3)$. [2]

(iv) $y = f(x) + 3$. [2]

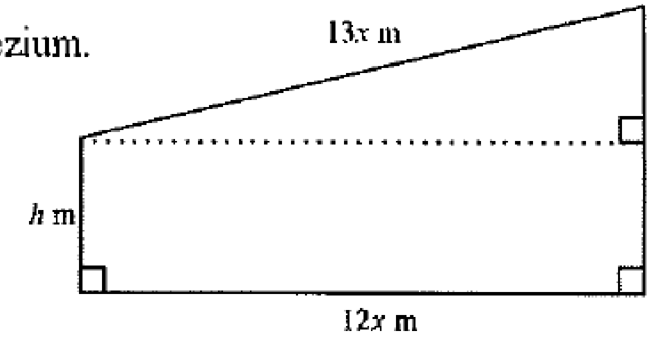
7. Given that $f(x) \equiv (5x^{-1} + 3x^{-2})^2 - 4$ where $x > 0$,

(i) find the value of x for which $f(x) = 0$. [6]

(ii) Find an equation of the tangent to the curve $y = f(x)$ at the point where $x = 1$. [6]

8. The diagram shows a plot of land in the shape of a trapezium.

   The perimeter of the plot of land is 200 m.



   (i) Show that $h = 100 - 15x$. [3]

   (ii) Show that the area of the plot is $150(a - (x - b)^2)$ m, where a and b are integers to be found. [6]

   (iii) Explain why this area is maximum when $x = b$, and hence state the largest area of the plot as x varies. [3]

9. The points A, B and C have coordinates $(-3, 4)$, $(7, 4)$ and $(5, 8)$ respectively.

   The straight lines l_1 and l_2 are the perpendicular bisectors of AB and BC respectively.

   (i) Find equations of l_1 and l_2. [6]

   (ii) Find the coordinates of the point P where l_1 and l_2 intersect. [3]

   (iii) Hence find an equation of the circle which passes through A, B and C. [3]

(iii) Hence find an equation of the circle which passes through A, B and C. [3]

CORE MATHS 1 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1.	$\dfrac{3\sqrt{2}-3(2+2\sqrt{2})}{\sqrt{2}(2+2\sqrt{2})} = \dfrac{-3\sqrt{2}-6}{2\sqrt{2}+4} = \dfrac{-3(\sqrt{2}+2)}{2(\sqrt{2}+2)} = -\dfrac{3}{2}$	M1 A1 M1 A1	4
2.	(i) $d/dx\ (6-5x-6x^2) = -5-12x$	B1 M1 A1	
	(ii) $d/dx\ (x^{3/2}) = \dfrac{3}{2}x^{1/2} = \dfrac{3}{2}\sqrt{x}$	M1 A1	5
3.	(i) $4/2 \times (t-6)/8 = -1$ $\quad t-6=-4 \quad t=2$	M1 A1 A1	
	(ii) $PR = 10$, height $= 4$ so area $= 20$	B1 M1 A1	6
4.	(i) $18/y^4 = 18x^2$	B1	
	(ii) $18x^2+x-4=0 \quad (2x+1)(9x-4)=0$	M1 A1 A1	
	$x = -1/2$ or $4/9$ $\quad$ Must have $x>0$, so $y = 3/2$	M1 A1	6
5.	Line meets curve where $9-x^2 = 1-2x \quad x^2-2x-8=0$	M1 A1	
	$(x+2)(x-4)=0 \quad x=-2, x=4$	M1 A1	
	Gradient of curve $= -2x = 4, -8$ at these points, $\neq$ ½ so not a normal	M1 A1 A1	7
6.	(i) Minimum at $(-3, 0)$, cuts axes at $(-3, 0)$ and $(0, 9)$	B2	
	(ii) Minimum at $(-3, 0)$, cuts axes at $(-3, 0)$ and $(0, 18)$	B2	

	(iii) Minimum at (0, 0), cuts axes only at (0, 0)	B2	
	(iv) Minimum at (–3, 3), cuts axes at (0, 12)	B2	8
7.	(i) $f(x) = 0$ when $\frac{5}{x} + \frac{3}{x^2} = \pm 2$ $2x^2 - 5x - 3 = 0$ or $2x^2 + 5x + 3 = 0$	M1 A1 A1	
	$(2x + 1)(x - 3) = 0$ or $(x + 1)(2x + 3) = 0$ $x > 0$, so $x = 3$	M1 A1 A1	
	(ii) $f(x) = 25x^{-2} + 30x^{-3} + 9x^{-4} - 4$ so $f'(x) = -50x^{-3} - 90x^{-4} - 36x^{-5}$	B1 M1 A1	
	At $x = 1$, $f(x) = 60$ and $f'(x) = -176$ $y - 60 = -176(x - 1)$	B1 M1 A1	12
8.	(i) Height of triangle = $5x$ (5, 12, 13), so $2h + 30x = 200$ $h = 100 - 15x$	B1 M1 A1	
	(ii) Area = $30x^2 + 12x(100 - 15x) = 1200x - 150x^2 = 150(8x - x^2)$	M1 A1 A1	
	$= 150(16 - (x - 4)^2)$ $a = 16$, $b = 4$	M1 A1 A1	
	(iii) $(x - 4)^2 = 0$ when $x = 4$ and > 0 otherwise, so area is max. at 2400 m^2	B1 M1 A1	12
9.	(i) l_1 is $x = 2$ Mid-point of BC is (6, 6)	B2 B1	
	Gradient of $BC = -2$ so l_2 is $y - 6 = \frac{1}{2}(x - 6)$	B1 M1 A1	
	(ii) $y = \frac{1}{2}(-4) + 6 = 4$ $P = (2, 4)$	M1 A1 A1	
	(iii) Radius = 5 $(x - 2)^2 + (y - 4)^2 = 25$	B1 M1 A1	12

